Depreciation of Business R&D Capital

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Abstract

We develop a forward-looking profit model to estimate the depreciation rates of business R&D capital. By using data from Compustat, BEA, and NSF between 1987 and 2008, and the newly developed model, we estimate both constant and time-varying industry-specific R&D depreciation rates. The estimates are the first complete set of R&D depreciation rates for major U.S. high-tech industries. They align with the main conclusions from recent studies that the rates are in general higher than the traditionally assumed 15 percent and vary across industries. The relative ranking of the constant R&D depreciation rates among industries is consistent with industry observations and the industry-specific time-varying rates are informative about the dynamics of technological change and the levels of competition across industries. Lastly, we also present a cross-country comparison of the R&D depreciation rates between the U.S. and Japan, and find that the results reflect the relative technological competitiveness in key industries.

JEL Codes: O30, C20, C80

Keywords: Depreciation, Research and Development, Technological Change

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1. Introduction

In an increasingly knowledge-based U.S. economy, measuring intangible assets, including research and development (R&D) assets, is critical to obtaining a complete picture of the economy and explaining its sources of growth. Corrado *et al.* (2007) pointed out that after 1995 intangible assets reached parity with tangible assets as a source of growth. Despite the increasing impact of intangible assets on economic growth, it is difficult to capitalize intangible assets in the national income and product accounts (NIPAs) and therefore to measure their impacts on economic growth. The difficulties arise because the capitalization involves several critical but difficult measurement issues. One of these is the measurement of the depreciation rate of intangible assets, including R&D assets.

The depreciation rate of R&D assets is required for capitalizing R&D investments in the NIPAs for two reasons. First, the depreciation rate is needed to construct knowledge stocks – it is the only asset-specific element in the commonly adopted user cost formula. This user cost formula is used to calculate the flow of capital services (Jorgenson, 1963, Hall and Jorgenson, 1967, Corrado *et al.*, 2007, Aizcorbe *et al.*, 2009), which is important for examining how R&D capital affects the productivity growth of the U.S. economy (Okubo *et al.*, 2006). Second, the depreciation rate is required in order to measure the rate of return to R&D (Hall, 2005).

As Griliches (1996) concludes, the measurement of R&D depreciation is the central unresolved problem in the measurement of the rate of return to R&D. The problem arises from the fact that both the price and output of R&D capital are generally unobservable. Additionally, there is no arms-length market for most R&D assets and the majority of R&D capital is developed for own use by the firms. Therefore it is difficult to independently compute the depreciation rate of R&D capital (Hall, 2005, Corrado *et al.*, 2007). Moreover, unlike tangible capital which depreciates partly due to physical decay or wear and tear, R&D capital depreciates mainly because its contribution to a firm's profit declines over time. The driving forces are obsolescence and competition (Hall, 2005), both of which reflect individual industry technological and competitive environments. Given that these environments can vary immensely across industries and over time, the resulting (private) R&D depreciation rates should also vary across industries and over time.

In response to these measurement difficulties, previous research has adopted four major approaches to calculate R&D depreciation rates: patent renewal, production function, amortization, and market valuation (Mead, 2007). As summarized by Mead (2007), all approaches encounter the problem of insufficient data on variation and thus cannot separately identify R&D depreciation rates without imposing strong identifying assumptions. Given the fact that firms' propensities to patent vary across industries and technology areas, the patent renewal approach cannot capture all innovation activities (Hall *et al.*, 2014). Moreover, innovations may remain valuable even if their patents have expired, given the other ways in which firms capture returns to R&D (Levin *et al.,* 1987). The patent renewal approach also suffers from the failure to observe the right hand tail of a very skewed value distribution due to the relatively low level of renewal fees. The identification problem can be mitigated by using citation-weighted patent data, but there is a truncation bias problem arising due to an incomplete observed citation life of patents (Hall *et al.*, 2000).

Using the production function and market value approaches has the advantage of incorporating all R&D rather than just that which is patented. However, these approaches generally rely on the assumption that the average realized rate of return is the same as the expected rate of return (Hall, 2005). This assumption allows one to back out the depreciation rate which makes the two consistent. We use a similar approach here, in that we assume a normal rate of return to R&D when computing the profit function.

An additional complication is the question of a gestation lag for the output of R&D. Most earlier research has failed to deal with the issue of gestation lags by treating them as zero or one year to calculate the R&D capital stock (Corrado *et al.*, 2007, but see Hall and Hayashi, 1989 for an exception). Because the product development life cycle varies across industries, this treatment is questionable for R&D assets so we explore the use of a gestation lag here.

This paper introduces a new approach by developing a forward-looking profit model that can be used to calculate both constant and time-varying industry-specific R&D depreciation rates. The model is built on the core concept that R&D capital depreciates because its contribution to a firm's profit declines over time. Our forward-looking profit model rests on some relatively simple assumptions that are plausible given the nature of the data and allows us to estimate R&D depreciation rates by using only data on R&D investment and sales or industry output.

The model is applied to two different datasets, one for firms and one for industries, to calculate constant R&D depreciation rates for all ten R&D intensive industries identified in BEA's R&D Satellite Account (R&DSA). The first dataset is constructed from Compustat SIC-based firm-level sales and R&D investments in ten R&D intensive industries. The second dataset contains BEA-NSF NAICS-based establishment-level industry output and R&D investments in ten R&D intensive industries. Both sets of estimates show that the derived R&D depreciation rates align with the major conclusions from recent studies that the rates should be higher than the traditional assumption (15 percent) and vary across industries.

Our new method demonstrates the feasibility of estimating R&D depreciation rates from industry data. Given that the BEA-NSF dataset better represents the industry population because it is not confined to publicly traded firms, we also apply the model to the BEA-NSF dataset to estimate the industry-specific time-varying R&D deprecation rates for five selected R&D intensive industries. The results are in general consistent with industry observations on the pace of technological change or reflect the appropriability condition of its intellectual property.

The remainder of this paper is organized as follows. Section 2 sets out our new R&D investment model. Section 3 presents a firm and industry-level data analysis that assumes constant depreciation rates over time. Section 4 presents time-varying depreciation rates for five selected BEA's R&D intensive industries. Section 5 presents the first cross-country comparison of R&D depreciation rates between the U.S. and Japan for several key R&D intensive industries, and concluding remarks are given in Section 6.

2. Model

The premise of our model is that business R&D capital depreciates because its contribution to a firm's profit declines over time. R&D capital generates privately appropriable returns; thus, it depreciates when its appropriable return declines over time. The expected R&D depreciation rate is a necessary and important component of a firm's R&D investment model. A profit-maximizing firm will invest in R&D such that the expected marginal benefit equals the marginal cost. That is, in each period t, a firm will choose an R&D investment amount to maximize the net present value of the expected returns to R&D investment:

$$\max_{R_t} E_t[\pi_t] = -R_t + E_t\left[\sum_{j=0}^{\infty} \frac{q_{t+j+d} I(R_t)(1-\delta)^j}{(1+r)^{j+d}}\right] \qquad (1)$$

where R_t is the R&D investment amount in period t, q_t is the sales in period t, $I(R_t)$ is the increase in profit rate due to R&D investment, δ is the R&D depreciation rate, and r is the cost of capital. The parameter d is the gestation lag and is assumed to be an integer which is no less than 0. R&D investment in period t will contribute to the profits in later periods but at a geometrically declining rate. We assume that the sales q for periods later than t grows at a constant growth rate, g. That is, $q_{t+j} = q_t\left(1+g\right)^j$. This assumption is consistent with the fact that the output of most R&D intensive industries grows fairly smoothly over time (See Figures B-1 and B-2 in the appendices).

Place figure 1 here.

To resolve the issue that the prices of most R&D assets are generally unobservable, we define *I(R)* as a concave function:

$$I(R) = I_\Omega\left(1-\exp\left[\frac{-R}{\theta}\right]\right) \qquad (2)$$

with $I''(R) < 0$. $I'(\mathrm{R}) = I_\Omega \exp\left(\frac{-R}{\theta}\right) > 0$, and $I'(0) = \mathrm{I}_\Omega = \lim_{R\to\infty} I(R)$. Figure 1 depicts how the function I gradually increases asymptotically to I_Ω with R, the current-period R&D investment. The increase in profit rate due to R&D investments, $I'(R)$, has an upper bound at I_Ω when $R = 0$. This functional form has few parameters but nevertheless shows the desired concavity with respect to R. In this, our approach is similar to that adopted by Cohen and Klepper (1996), who show that when there are fixed costs to an R&D program and firms have multiple projects, the resulting R&D productivity will be heterogeneous across firms and self-selection will ensure that the observed productivity of R&D will vary negatively with firm size. Our model incorporates the assumption of diminishing marginal returns to R&D investment implied by their assumptions, which is more realistic than the traditional assumption of constant returns to scale (Griliches, 1996). In addition, the model implicitly assumes that innovation is incremental, which is appropriate for industry aggregate R&D, most of which is performed by large established firms.

The function I includes a parameter θ that defines the investment scale for increases in R&D and acts as a deflator to capture the increasing time trend of R&D investment as a component of investment in many industries. The value of θ can vary from industry to industry, allowing different R&D investment scales for different industries. In Figure B-3 and B-4, both the BEA-NSF industry data and the Compustat average firm data show that the average R&D investment in most industries increases greatly over a period of two decades, and therefore we expect that the investment scale, θ, needed to achieve the same increase in profit rate should grow accordingly.

Using this function for the profitability of R&D, the R&D investment model becomes the following:

$$\begin{aligned} E_t[\pi_t] &= -R_t + E_t\left[\sum_{j=0}^{\infty} \frac{q_{t+j+d} I(R_t)(1-\delta)^j}{(1+r)^{j+d}}\right] \\ &= -R_t + I_\Omega\left[1-\exp\left(-\frac{R_t}{\theta_t}\right)\right]\sum_{j=0}^{\infty}\frac{E_t[q_{t+j+d}](1-\delta)^j}{(1+r)^{j+d}} \end{aligned} \tag{3}$$

Note that we have assumed that *d, r,* and δ are known to the firm at time t. Because θ varies over time, we model the time-dependent feature of θ by $\theta_t \equiv \theta_0(1+G)^t$, where G is the growth rate of θ_t. To estimate G, we assume that the growth pattern of industry's R&D investment and its R&D investment scale are similar and we estimate G by fitting the data for R&D investment to the equation, $R_t = R_0(1+G)^t$. This approach is justified by the fact that BEA data on most industry R&D grows somewhat smoothly over time (See Figure B-3), Using this assumption, Equation (3) becomes:

$$\pi_t = -R_t + I_\Omega\left[1-\exp\left(-\frac{R_t}{\theta_0(1+G)^t}\right)\right]\frac{q_t(1+g)^d}{(1+r)^{d-1}(r+\delta-g+g\delta)} \tag{4}$$

Note that because of our assumptions of constant growth in sales and R&D, there is no longer any role for uncertainty in this equation, and therefore no error term. Assuming profit maximization, the optimal choice of Rt implies the following first order condition:

$$\frac{\partial \pi_t}{\partial R_t} = -\frac{(1+G)^t}{I_\Omega}\square\theta_0 \exp\left[\frac{R_t}{\theta_0(1+G)^t}\right] + \frac{q_t(1+g)^d}{(1+r)^{d-1}(r+\delta-g+g\delta)} = 0 \tag{5}$$

For estimation, we add a disturbance to this equation (reflecting the fact that it will not hold identically for all industries in all years) and then estimate θ_0 and the depreciation rate δ.

3. Estimation with constant R&D depreciation rates

As a first step in our empirical analysis, we estimate the time-constant R&D depreciation rates based on two datasets. One is the firm-level Compustat dataset from 1989 to 2008 and the other is the industry-level BEA-NSF dataset from 1987 to 2007. The Compustat dataset contains firm-level sales and R&D investments for ten SIC-based industries. Their corresponding SIC codes and numbers of firms are listed in Table 1. For the Compustat data, we take the average values of annual sales and R&D investment in each industry for estimation. If the number of firms was the same every year, using means would be the same as using aggregate data; however, there is entry and exit in this dataset so the results are based on average firm behavior. The BEA-NSF data that we use is fundamentally different, as it is designed to measure true industry aggregates (correcting for such things as firm presence in multiple industries, something we are unable to do with Compustat data).

Place table 1 here.

The model used for estimation, based on equation (5), is shown below:

$$\varepsilon_t \equiv \frac{\left(1+\hat{G}\right)^t}{I_\Omega} \cdot \theta_0 \exp\left[\frac{R_t}{\theta_0\left(1+\hat{G}\right)^t}\right] - \frac{q_t\left(1+\hat{g}\right)^d}{\left(1+r\right)^{d-1}\left(r+\delta-\hat{g}+\hat{g}\delta\right)} \tag{6}$$

Where $\hat{g}$ and $\hat{G}$ are estimated using the entire time period. In order to estimate, we need to make assumptions about I_Ω, r, and d. The value of I_Ω can be inferred from the BEA annual return rates of all assets for non-financial corporations. As Jorgenson and Griliches (1967) argue, in equilibrium the rates of return for all assets should be equal to ensure no arbitrage, and so we can use a common rate of return for both tangibles and intangibles (such as R&D assets). For simplicity, I_Ω is set to be the average return rates of all assets for non-financial corporations during 1987-2008, which is 8.9 percent. In addition, in equilibrium the rate of return should be equal to the cost of capital. Therefore, we use the same value for r. Later in the paper we perform a sensitivity analysis using time-varying rates of return, based both on the 3 month T-bill rate plus a risk adjustment of 4 per cent and on the BEA's own time-varying rate of return to assets.

We use a 2-year gestation lag d, which is consistent with the finding in Pakes and Schankerman (1984) who examined 49 manufacturing firms across industries and reported that gestation lags between 1.2 and 2.5 years were appropriate values to use (see also Hall and Hayashi, 1989). In addition, according to the recent U.S. R&D survey conducted by BEA, Census Bureau and National Science Foundation (NSF) in 2010, the average gestation lag is 1.94 years for all industries. [1] We also report estimates using a gestation lag of zero years.

R_t and q_t are taken from the data and also used to compute the average growth rates of output (G) and of R&D (g), so the only unknown parameters in the equation are δ and θ. Given these assumptions, δ and θ are estimated by nonlinear least squares (NLLS) and nonlinear generalized method of moments (GMM), using equation (6).

3.1 Nonlinear Least Squares Estimates

This section of the paper reports the results of NLLS estimation using our two datasets. Table 3 shows the two sets of estimated industry-specific constant R&D depreciation rates based on the Compustat company-based data and the BEA-NSF establishment-based data. With the exception of computer system design and motor vehicles, the rates estimated using BEA-NSF data tend to be lower than those using Compustat data, which might be consistent with spillovers at the industry level not captured by publicly traded firm data. The depreciation rates are consistent with most industry observations. For example, the pharmaceutical industry has the lowest R&D depreciation rates in both sets of estimates, which may reflect the fact that R&D resources in pharmaceuticals are more appropriable than in other industries due to effective patent protection and other entry barriers. Compared with the pharmaceutical industry, the computers and peripheral industry has a higher R&D depreciation rate, which is consistent with industry observations that the industry has adopted a higher degree of global outsourcing to source from few global suppliers (Li, 2008). Module design and efficient global supply chain management has made

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[1] The average gestation lag is based on the responses from 6,381 firms across 38 industries in the NSF 2010 Business R&D and Innovation Survey (BRDIS).

the products introduced in this industry more like commodities, which have shorter product life cycles.

Place tables 2 and 3 here.

Table 2 showed the time-constant R&D depreciation rates estimated by other recent studies. Comparing Table 3 with Table 2, we can see several key results from this study. First, the estimated industry-specific R&D depreciation rates are consistent with those of recent studies, which indicate that depreciation rates for business R&D are likely to vary across industries due to the different competition environments and paces of technology change. Second, most industries have R&D depreciation rates higher than the traditionally assumed 15 percent that has been the benchmark for much of the empirical work (Grilliches and Mairesse, 1984, Berstein and Mamuneas, 2006, Corrado *et al.,* 2007, Hall, 2007, Huang and Diewert, 2007, Warusawitharana, 2010). Third, the R&D depreciation rate in the scientific research and development industry is much higher than that in the pharmaceutical industry.[2] This is consistent with industry observations that in the past two decades, there has been little innovation in the traditional pharmaceutical industry and biopharmaceuticals has faster growth rate of innovation. For example, in 1988, only 5 proteins from genetically engineered cells had been approved as drugs by the U.S. FDA, but the number has skyrocketed to over 125 by the end of 1990s (Colwell, 2002).

Among the R&D depreciation rates in the ten analyzed R&D intensive industries, the values for the aerospace and auto industries are usually large compared to those for other industries. For example, the estimated R&D depreciation rates for the auto industry are 56.0 percent and 74.3 percent for the Compustat and BEA-NSF data respectively. These results are not inconsistent with the result of the UK's ONS (Office of National Statistics) survey of the R&D service lives (Haltiwanger *et al.*, 2010). The average R&D service life for the auto industry in the UK's ONS survey is 4.3 years, which implies an R&D depreciation rate over

[2] According to NSF's BRDIS in 2009, biotech firms account for over 65% of R&D investments in the scientific research and development industry. Other firms related to physical, engineering, and life sciences account for around 34.5% of R&D investments.

40 percent. Note that the response rate of the UK's ONS survey, however, is reported to be low.[3]

In our formulation of the R&D investment model, there is an implicit tradeoff between the assumed *ex ante* rate of return and the computed depreciation rate. Essentially the depreciation rate for private business R&D is determined by the competitive environment of the firms that do it, and if the rate of return turns out to be lower than expected, the implication is that the value of the R&D has depreciated. We illustrate this tradeoff by reestimating our model for the aerospace and auto industries with an assumed rate of return to R&D of 1 percent. This is justified by two facts: First, the U.S. auto industry had negative return rates during the data period.[4] Second, in its August 2011 report on the Aerospace and Defense industrial base assessments, the Office of Technology Evaluation at Department of Commerce reports that the industry's profit margin is around 1% and may be only 10% of the performance of high-tech industries in Silicon Valley (Department of Commerce, 2011).[5]

Table 3 reports estimates for these two industries that use the lower rates of return in italics and they are much lower, around 7-15 percent, confirming our intuition about the tradeoff between rates of return and depreciation. It is also worth noting that the data quality of R&D expenses in the auto and the aerospace industries are poor and the R&D data based on 10-K & 10-Q reports do not cover the industry well. For example, in the aerospace industry, some firms clearly report their own investment in R&D, but others report R&D expenses that combine federally funded and company-funded R&D.

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[3] In 2011 and 2012, the UK's ONS conducted two back-to-back surveys on 1701 firms and found a median R&D service life of 6 years for all industries. Compared with 2.1% in the U.S. similar survey in 2010, the two surveys have better response rates at around 43%. However, the survey result has a very high degree of uncertainty (Kerr, 2014; Li, 2014). For example, the average answer difference from the same correspondent for the same company is 3.9 years and the average difference from different correspondents is 4.5 years. The UK's survey result is consistent with the U.S.'s finding that most respondents could not answer questions related to the R&D service lives correctly (Li, 2012). In the end, the UK's ONS adopts 16% as the R&D depreciation rate for all industries.

[4] Private communication with Brian Sliker at BEA, an expert in the return rate of industry assets, confirmed this negative trend in the auto industry.

[5] After using the new modified model, our new estimate is 29% higher than the rate in Huang and Diewert (2007). However, in the later section of cross-country comparison, the estimates between the U.S. and Japan in this industry are reasonable. Diewert reports in private communication that they found computing the optimal rate in this sector difficult.

Table 4 presents the results of a sensitivity analysis for the gestation lag and *ex ante* rate of return. The first two sets of columns compare gestation lags of two and zero years.[6] In general, the estimated depreciation rates do not differ a great deal, and those for the zero lag are slightly higher, except in the software, computer system design, and scientific research and development. Interestingly, these three sectors are the only service sectors. A possible interpretation of the general result is the following: if the gestation lag is zero rather than two, effectively there is a greater stock of R&D over which to spread the same profits, so it must depreciate more rapidly to explain the same rate of return. The fact that the service sectors do not follow this pattern is somewhat puzzling but is doubtless due to the specific trends in R&D and output in those sectors.

Place table 4 here.

The estimates are not sensitive to allowing a variable cost of capital (although as we saw earlier, they are sensitive to a change in the overall level. The last two sets of columns in Table 4 show results when the cost of capital/rate of return is set to (1) the riskfree 3-month treasury bill rate plus a risk premium of 4 percent or (2) BEA's own measured average rate of return to assets during the year. Figure B-5 displays these time series. There is little difference in the estimates across these columns. Figure 2 graphs the sensitivity of the estimated depreciation rate of R&D assets to the assumed cost of capital for each industry separately. There are clear differences across the industries, with autos, computer hardware and services, aerospace, and instruments the most sensitive to the assumption, and the other sectors much less sensitive.

Place figure 2 here.

3.2 Nonlinear GMM

We may be concerned that simultaneity between current output and R&D (due to cash flow or demand shocks) could bias estimates of the relation in equation (6). To check this possibility we estimated the equation using nonlinear GMM, choosing lagged values of R&D and output as instruments. The choice of instrument variables is based on the assumption

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[6] BEA adopts a zero gestation lag, on the grounds that when a firm invests in R&D, the R&D investment should contribute immediately to the firm's knowledge stock.

that (given a forward-looking profit model) previous R&D investments and output are not related to any shocks (ε) to the optimal R&D plan described by equation (6).

Place table 5 here.

Table 5 compares the estimates based on nonlinear least squares and nonlinear GMM, both computed with a two-year gestation lag and an expected rate of return equal to 8.9%. In general, the nonlinear GMM estimates have higher standard errors than those associated with the nonlinear least squares estimates, although not always. With the exception of the aerospace sector, where the estimated depreciation rate is much lower, the estimates are very similar to those obtained using nonlinear least squares. We also report the results of a test of the over-identifying restriction (degrees of freedom equal to one), which passes only for the aerospace and motor vehicle sector. If future datasets are larger in size and we are able to find better instruments, the nonlinear GMM approach might provide a more robust estimation, but for the current data these results suggest that the nonlinear least squares estimates are adequate.

4. Estimation with time-varying R&D depreciation rates

Since the technological and competition environments change over time, the R&D depreciation rates are expected to vary through the 21 years of data studied. Therefore, there is a need to calculate industry-specific and time-dependent R&D depreciation rates. We use the same industry output and R&D investment data from the BEA-NSF dataset. Unlike the Compustat dataset which contains only the data of large publicly traded firms, the BEA-NSF data better represent the industry by including firms with 5 or more employees.[7] The time-dependent feature of δ was obtained by minimizing Equation (6) with subsets of data. Instead of using all years of data, we performed least squares fitting over a five-year interval each time, with a step of 2 years in progression. As a result, the data-model fit is carried out nine times for 21 years of data, and each estimated depreciation rate is assigned to the center of a time window. The values of d, I_Ω, and r are defined in the same manner as before. Although there are only 5 data points to estimate the two parameters, the

[7] The R&D data come from the NSF's BRDIS. BRDIS is a nationally representative sample of all companies with 5 or more employees in all industries.

estimates generally converged well and the standard error estimates are not that large, except in a few cases.

Place figure 3 here.

Figure 3 shows the best-fit time-varying R&D depreciation rates for all ten industries together with their standard errors; the figures are plotted on the same scale to facilitate comparison. The industries differ in their volatility considerably, with software, pharmaceuticals, semiconductors, motor vehicles and scientific R&D being relatively stable, whereas the industries strongly affected by hardware-related technical change during this time period are much more volatile (e.g., computing equipment, aerospace, communication equipment, computer system design, and scientific instruments). One concern with these results may be the underlying data: industries like semiconductors and motor vehicles whose R&D is dominated by very large firms may be somewhat better measured than the communication equipment or instruments sector.

Place figure 4 here.

Figure 4 shows the results of a similar estimation using the sales and R&D of the average firm on Compustat for comparison. It is important to keep in mind that these series will be very different in some cases but not in others. For example, pharmaceutical R&D (as opposed to biotechnology R&D) is largely conducted in firms assigned by Compustat to the pharmaceutical sector, so the BEA and Compustat data will be similar. In contrast, most software R&D is conducted by hardware firms, so that if when we compute the average R&D and sales in the software sector as defined by Compustat, it is very different from the software R&D spending and output measured by BEA and NSF, which includes that from hardware firms. Similar statements might be made about computer systems design and we do see that the trends in both these sectors are very different looking across results from the two datasets in Figures 3 and 4. In contrast, the depreciation rates in pharmaceuticals look fairly similar, and are generally around 15-20 per cent.

Figures 3 and 4 also reveal some other facts about the industries we studied. First, the pharmaceutical industry has a somewhat declining depreciation pattern, which implies a slower pace of technological change. This is consistent with the industry's consensus that factors such as stricter FDA approval guidelines have negatively affected the industry's productivity growth in R&D in recent years. As a result, the industry has been experiencing a negative productivity growth in R&D in recent years. For example, during the period of 1990 to 1999, the FDA approved an average of 31 drugs per year, but this number dropped to 24 during the period of 2000 to 2009 (Rockoff and Winslow, 2011) and further went down to 21 in 2010 (Lamattina, 2011). However, the scientific R&D industry, which is composed majority by biotech firms, has a higher level of depreciation rates that has not declined since 1990. This echoes the fact mentioned previously that, in the past two decades, there has been little innovation in the traditional pharmaceutical industry and the biopharmaceuticals industry has faster growth rate of innovation.

Second, the R&D depreciation rate of the semiconductor industry shows a clear declining trend after 2000 in both datasets, albeit imprecisely measured. This depreciation pattern is consistent with several research results. For example, since 2000, the rate of technological change in the microprocessor industry has slowed (Flamm, 2007). By combining our depreciation pattern with the evidence of a slower pace of productivity growth in the semiconductor industry after 2000 (Jorgenson *et al.*, 2012), we find that our result supports Jorgenson's hypothesis (2001) that the increase in the pace of technological change in this sector is positively related to faster productivity growth.

Third, the computer and peripherals equipment industry had stable R&D depreciation before 1995, a decline during the late 1990s, and then increased slowly or stabilized after 2001. It is helpful to recall the result by Hall (2005) who shows a pattern of decreasing depreciation for the computers, communication equipment, and scientific instrument industries during the period of 1989 to 2003. Since Hall's result is based on the data including two additional high-tech industries, it is not adequate to directly compare the depreciation patterns between the two studies. Nonetheless, it is well known that since the late 1990s, the products in the computer and peripherals equipment industry have also become more like commodities,[8] a trend that implies a shorter product life cycle, a higher degree of market competition, and possibly a slower pace of technological change, mirroring that in semiconductors.

Lastly, the R&D depreciation of the software industry, as measured by the BEA-NSF data, also experienced a declining trend during the period from 1995 to early 2000s. The

[8] Note that in recent years the International Consumer Electronics Show (CES) has become more important than ever for the computer manufacturers to introduce their new products and prototypes.

declining trend reflects the fact that, compared with the variable technology environment during the period from 1980s to early 1990s, the Wintel system provided a more stable development environment starting from mid-1990s.

5. Cross-country Comparison: U.S. vs. Japan

The R&D depreciation rate is one of the critical elements in computing R&D stock for the analysis of a country's productivity and economic growth. At the present time, however, there is no consistent methodology to estimate industry-specific R&D depreciation rates across countries. When no survey and/or research information is available, Eurostat recommends that a single average service life of 10 years should be retained (Eurostat, 2012). As a result, many OECD countries adopted R&D depreciation rates close to either Eurostat's recommendation or the traditional assumed 15 percent. The lack of variations in R&D depreciation rates across countries and across industries implies that countries, no matter in technology frontier or not, have a similar pace of technological progress and degree of market competition across countries. This result contradicts existing trade and growth theories.

Our method is an attempt to provide a consistent and reliable way to estimate industry-specific R&D depreciation rates across countries and to enable cross-country comparisons. Table 6 compares the estimated R&D depreciation rates between the U.S. and Japan for all industries that are R&D-intensive in Japan, including the drugs and medicines industry, the electrical machinery, equipment and supplies industry, the information and communication electronic equipment industry, and the transportation equipment. Due to data availability limitations, the estimates of Japanese R&D depreciation rates cover the period of 2002 to 2012. All estimates are based on a 2-year gestation lag, and the values of I_{Ω} and r are assumed to be 0.06, a number that is provided by Japan's National Accounts Department.

Place table 6 here.

Table 6 shows several important results. First, for the information and communication electronic equipment industry, the R&D depreciation rates between the two countries are very similar, after considering the standard errors. Second, compared with the counterpart in Japan, the U.S. pharmaceutical industry has a slightly smaller R&D depreciation rate, implying that U.S. pharmaceutical firms have a slight technology edge in this field and can better appropriate the returns from their investments in R&D assets. This result is consistent with the U.S. International Trade Commission's report on the global medical device industry, where it finds that, in terms of technological advantage, the U.S. is ranked first in the world and Japan is a close second (USITC, 2007). Third, Japan's lower R&D depreciation rate in the auto and electrical machinery industries indicates that, in these two industries, Japan has a clear technological edge and can better appropriate the return from its investments in R&D. This is also consistent with the industry observations on those two major industries.

6. Conclusions

R&D depreciation rates are critical to calculating the rates of return to R&D investments and capital service costs, which are important for capitalizing R&D investments in the national income accounts. Although important, measuring R&D depreciation rates is extremely difficult because both the price and output of R&D capital are generally unobservable.

In this research, we developed a forward-looking profit model to derive industry-specific R&D depreciation rates. Our model uses only data on R&D and output together with a few simple assumptions on the role of R&D in generating profits for the firm. Using some plausible assumptions about the expected rate of return for R&D, the model allows us to calculate not only industry-specific constant R&D depreciation rates but also time-varying rates.

We used both nonlinear least squares and nonlinear GMM to fit the model to the data. Both gave similar results, although GMM passed the overidentification test only part of the time. Future work would be useful to find better instruments and to improve the quality of the underlying data.

Our research results highlight several promising features of the new forward-looking profit model: First, the derived constant industry-specific R&D depreciation rates are consistent with the conclusions from recent studies that depreciation rates for business R&D are likely to be more variable due to different competition environments across industries and higher than traditional 15 percent assumption (Bernstein and Mamuneas, 2006, Corrado et al., 2007, Hall, 2005, Huang and Diewert, 2007, Warusawitharana 2010). Second, the time-varying results capture the heterogeneous nature of industry environments in technology and competition. For example, by combining our depreciation pattern in the semiconductor industry with the evidence of a slower pace of productivity growth in the same industry after 2000 (Jorgenson *et al.*, 2012), we find support to Jorgenson's hypothesis (2001) that the increase in the pace of technological change is positively related to faster productivity growth. Third, our method provides a consistent and reliable way to perform cross-country comparisons of R&D depreciation rates, which can inform countries' relative paces of technological progress and technological environments as exemplified in the U.S.-Japan comparison. Lastly, it is well known that no direct measurements can verify any estimate of R&D depreciation rates. However, our results are consistent with the observations for many industries and across countries.

While this study provides the first complete set of industry-specific business R&D depreciation rates for all ten R&D intensive industries identified in BEA's R&D Satellite Account, future research can make improvements in several areas. In future research, we can modify the model to relax the assumption. First, current estimation uses nominal R&D and output data. When the industry-specific price index of R&D assets becomes available, we can improve the estimates by explicitly incorporating price level change. Second, the current model assumes the decision maker has a perfect foresight. Future research can relax this assumption by including the uncertainty in the model. Lastly, the current model assumes decreasing marginal returns to R&D investments and innovations to be incremental. Future research may relax these two assumptions and modify the model to be applicable to the industry with increasing marginal returns to R&D investments and drastic innovations.

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Table 1: Industry, Correspondent SIC Codes, and Numbers of Firms

Industry	*SIC Codes*	*Firms*	*Observations*
Computers and peripheral equipment	3570-3579, 3680-3689, 3695	395	2,398
Software	7372	1,041	7,132
Pharmaceuticals	2830, 2831, 2833-2836	1,046	8,531
Semiconductor	3661-3666, 3669-3679	876	6,132
Aerospace product and parts	3720, 3721, 3724, 3728, 3760	109	951
Communication equipment	3576, 3661, 3663, 3669, 3679	596	4,277
Computer system design	7370, 7371, 7373	471	2,519
Motor vehicles, bodies and trailers, and parts	3585, 3711, 3713-3716	308	1,639
Navigational, measuring, electromedical, and control instruments	3812, 3822, 3823, 3825, 3826, 3829, 3842, 3844, 3845	887	6,489
Scientific research and development	8731	118	598

Notes:
1. SIC codes containing only few firms are not listed.
2. The data are an unbalanced panel with 40,666 observations on about 5,847 firms between 1989 and 2008, drawn from Compustat.

Table 2: Summary of Previous Studies on R&D Depreciation Rates

Study	*Industry*	*Estimate*	*Method*	*Data*
Lev and Sougiannis (1996)	Chemicals	11%	Amortization	825 U.S. firms over the period of 1975-1991
	Electrical equipment	13%		
	Industrial machinery	14%		
	Scientific instruments	20%		
	Transportation equipment	14%		
Ballester et al. (2003)	Chemicals	14%	Amortization	652 U.S. firms over the period of 1985-2001
	Electrical equipment	13%		
	Industrial machinery	14%		
	Scientific instruments	14%		
	Transportation equipment	17%		
Knott et al. (2003)	Pharmaceuticals	88-100%	Production function	40 U.S. firms over the period of 1979 -1998
Berstein and Mamuneas (2006)	Chemicals	18%	Production function	U.S. industries over the period of 1954-2000
	Electrical equipment	29%		
	Industrial machinery	26%		
	Transportation equipment	17%		
Hall (2005)	Computers and scientific instruments	-5%	Production function	16750 U.S. firms over the period of 1974-2003
	Electrical equipment	-3%		
	Chemicals	-2%		
	Drugs and medical instruments	-11%		
	Metal and machinery	-2%		
Hall (2005)	Computers and scientific instruments	31%	Market valuation	16750 U.S. firms over the period of 1974-2003
	Electrical equipment	36%		
	Chemicals	19%		
	Drugs and medical instruments	15%		
	Metal and machinery	32%		
Huang and Diewert (2007)	Chemicals	1%	Production function	U.S. industries over the period of 1953-2001
	Electrical equipment	14%		
	Industrial machinery	3%		
	Transportation equipment	27%		
Warusawitharana (2010)	Semiconudctors	34%	Market valuation	U.S. industries over the period of 1987-2006
	Computer hardware	28%		
	Medical equipment	37%		
	Pharmaceuticals	41%		
	Software	37%		

Note: With the exception of Berstein and Mamuneas (2006) and Huang and Diewert (2007), all of the studies are based on US Computstat data.

Table 3: Nonlinear Least Squares estimates of the R&D depreciation rate

	Compustat Data		*BEA-NSF Data*	
Time period	*1989-2008*		*1987-2007*	
Industry	*Estimate*	*s.e.*	*Estimate*	*s.e.*
Computers and peripheral equipment	53.5%	2.1%	36.3%	3.8%
Software	35.0%	0.6%	30.8%	0.5%
Pharmaceutical	15.1%	1.0%	11.2%	4.8%
Semiconductor	29.3%	1.1%	22.6%	3.7%
Aerospace products and parts	88.0%	1.3%	33.9%	6.5%
Aerospace products and parts with ROR = 1%	*15.9%*	*0.2%*	*6.3%*	*0.6%*
Communication equipment	30.8%	1.2%	19.2%	3.3%
Computer system design	31.4%	3.0%	48.9%	7.9%
Motor vehicles, bodies and trailers, and parts	56.0%	2.3%	73.3%	2.9%
Motor vehicles, bodies and trailers, and parts, with ROR = 1%	*11.0%*	*0.3%*	*11.9%*	*0.4%*
Navigational, measuring, electromedical, and control instruments	47.2%	1.3%	32.9%	7.4%
Scientific research and development	32.6%	2.2%	29.5%	2.6%

Note: Gestation lag is 2 years; ex ante rate of return is 8.9

Table 4: Sensitivity of the depreciation rate to assumptions – BEA-NSF data

Gestation lag in years	*0*		*2*		*2*		*2*	
Interest rate	*8.9%*		*8.9%*		*Tbill + 4%*		*BEA return*	
	Estimate	*s.e.*	*Estimate*	*s.e.*	*Estimate*	*s.e.*	*Estimate*	*s.e.*
Computers and peripheral equipment	42.8%	4.3%	36.3%	3.8%	36.0%	4.4%	35.8%	4.5%
Software	28.8%	0.4%	30.8%	0.5%	32.3%	0.5%	30.6%	0.7%
Pharmaceutical	11.5%	4.9%	11.2%	4.8%	13.1%	3.1%	11.8%	3.7%
Semiconductor	25.1%	4.0%	22.6%	3.7%	23.1%	3.4%	22.4%	4.0%
Aerospace	39.0%	7.3%	33.9%	6.5%	32.4%	7.4%	35.0%	7.2%
Communication equipment	22.2%	3.7%	19.2%	3.3%	18.9%	2.9%	18.9%	2.9%
Computer system design	47.1%	7.6%	48.9%	7.9%	49.8%	6.6%	49.4%	6.7%
Motor vehicles, bodies and trailers, and parts	81.9%	3.2%	73.3%	2.9%	74.8%	3.6%	73.0%	2.9%
Navigational, measuring, electromedical, & control instruments	37.1%	8.2%	32.9%	7.4%	32.7%	7.8%	33.0%	6.8%
Scientific research and development	29.3%	2.6%	29.5%	2.6%	32.6%	3.1%	31.4%	2.3%

Method of estimation is nonlinear least squares.

Table 5: Comparing estimation methods

Industry	NLLS			NL GMM			
	Estimate	*s.e.*		*Estimate*	*s.e.*	*Test#*	
Computers and peripheral equipment	36.3%	3.8%		36.9%	2.7%	0.057	*
Software	30.8%	0.5%		30.4%	0.8%	0.052	*
Pharmaceutical	11.2%	4.8%		13.7%	3.9%	0.002	***
Semiconductors	22.6%	3.7%		23.7%	5.5%	0.006	***
Aerospace products and parts	33.9%	6.5%		9.1%	9.2%	0.543	
Communication equipment	19.2%	3.3%		23.8%	5.2%	0.000	***
Computer system design	48.9%	7.9%		47.6%	21.3%	0.000	***
Motor vehicles, bodies and trailers, and parts	73.3%	2.9%		63.9%	27.9%	0.192	
Navigational, measuring, electromedical, & control instruments	32.9%	7.4%		33.1%	10.1%	0.000	***
Scientific research and development	29.5%	2.6%		31.0%	2.6%	0.029	**

Notes:
Assumed gestation lag is two years; interest rate is 8.9%.
BEA-NSF data
Estimates shown are for the depreciation rate and its standard error.
The p-value of a test for overidentifying restrictions is reported in these columns.

Table 6: Comparing estimates for US and Japan

Industry	US		Japan	
	Estimate	*s.e.*	*Estimate*	*s.e.*
(1) Drugs and medicines	11.2%	4.8%	13.6%	0.7%
(2) Electrical machinery, equipment, and supplies	19.2%	3.3%	26.0%	4.3%
(3) Information and communication electronic equipment	32.9%	7.4%	23.5%	1.2%
(4) Transportation equipment	73.3%	2.9%	20.2%	0.6%

Notes:

1. The estimates are based on a 2-year gestation lag.
2. The U.S. data cover the period of 1987 to 2007 and the Japan's data cover the period of 2002 to 2012.
3. The corresponding industries in the U.S. are: (1) the pharmaceutical industry, (2) the communication equipment industry, (3) the navigational, measuring, electromedical, and control instruments industry, and (4) the motor vehicles, bodies and trailers, and parts industry.

Figure 1: The Concavity of $I(RD)$

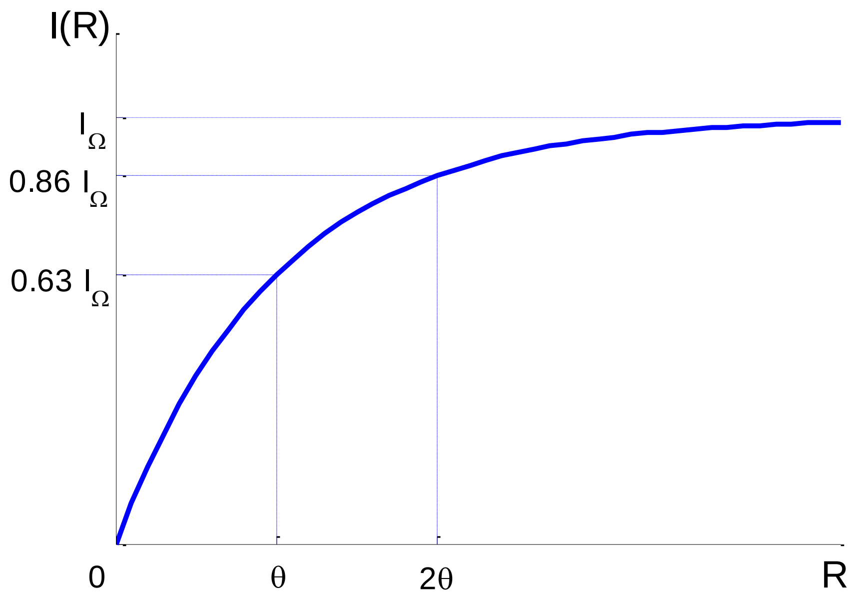


Figure 2: Sensitivity of the depreciation rate of R&D Assets to the Cost of Capital

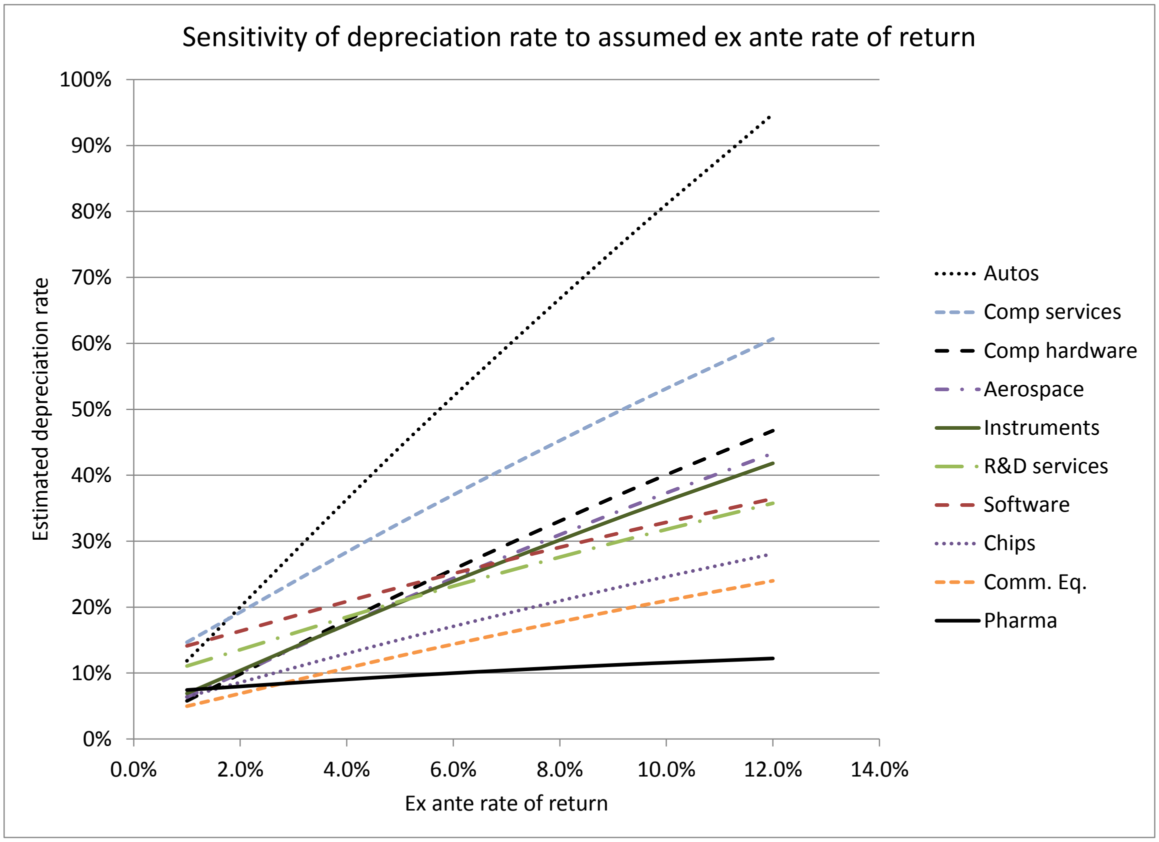


Figure 3: Time-varying R&D Depreciation Rates Based on BEA-NSF data

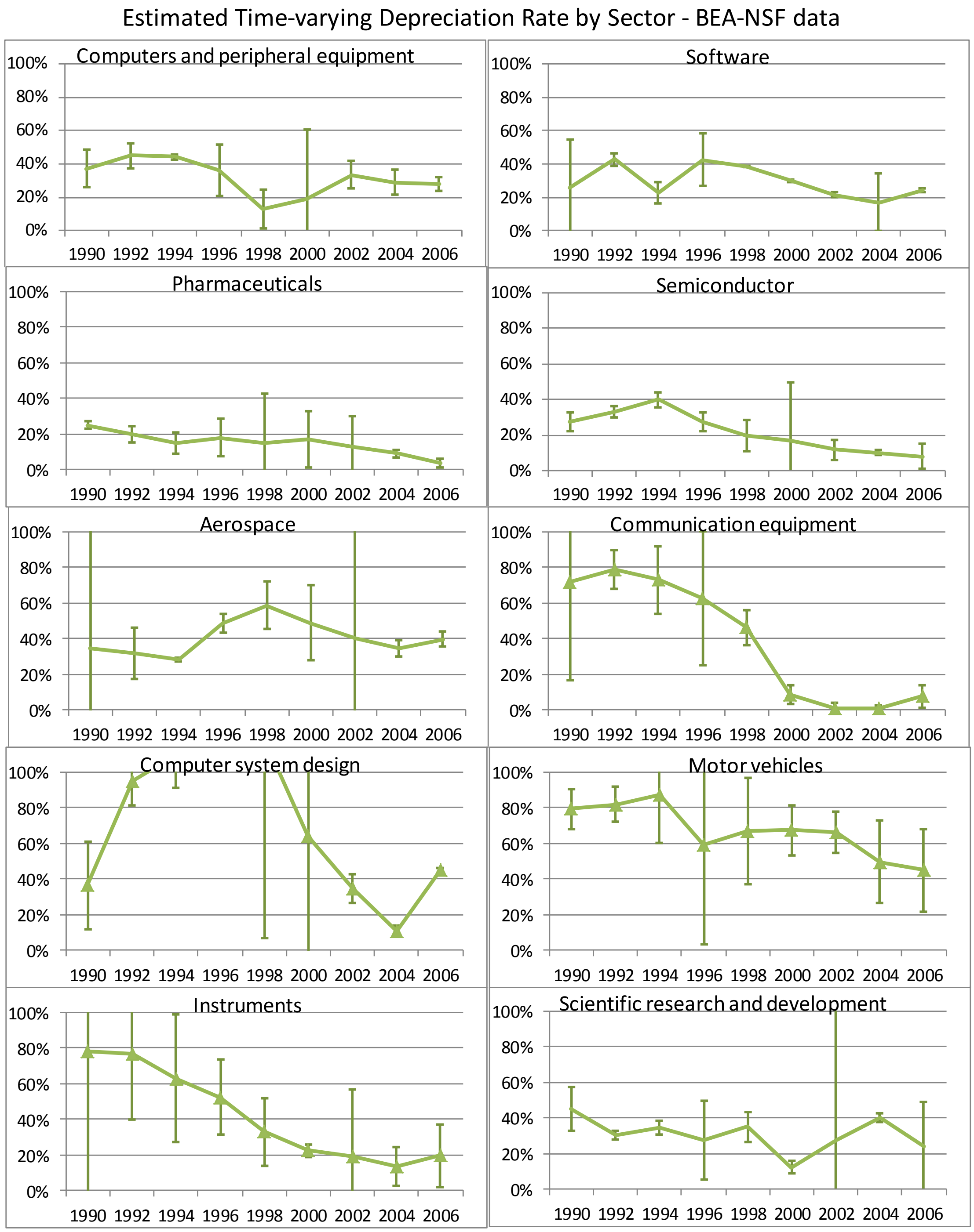
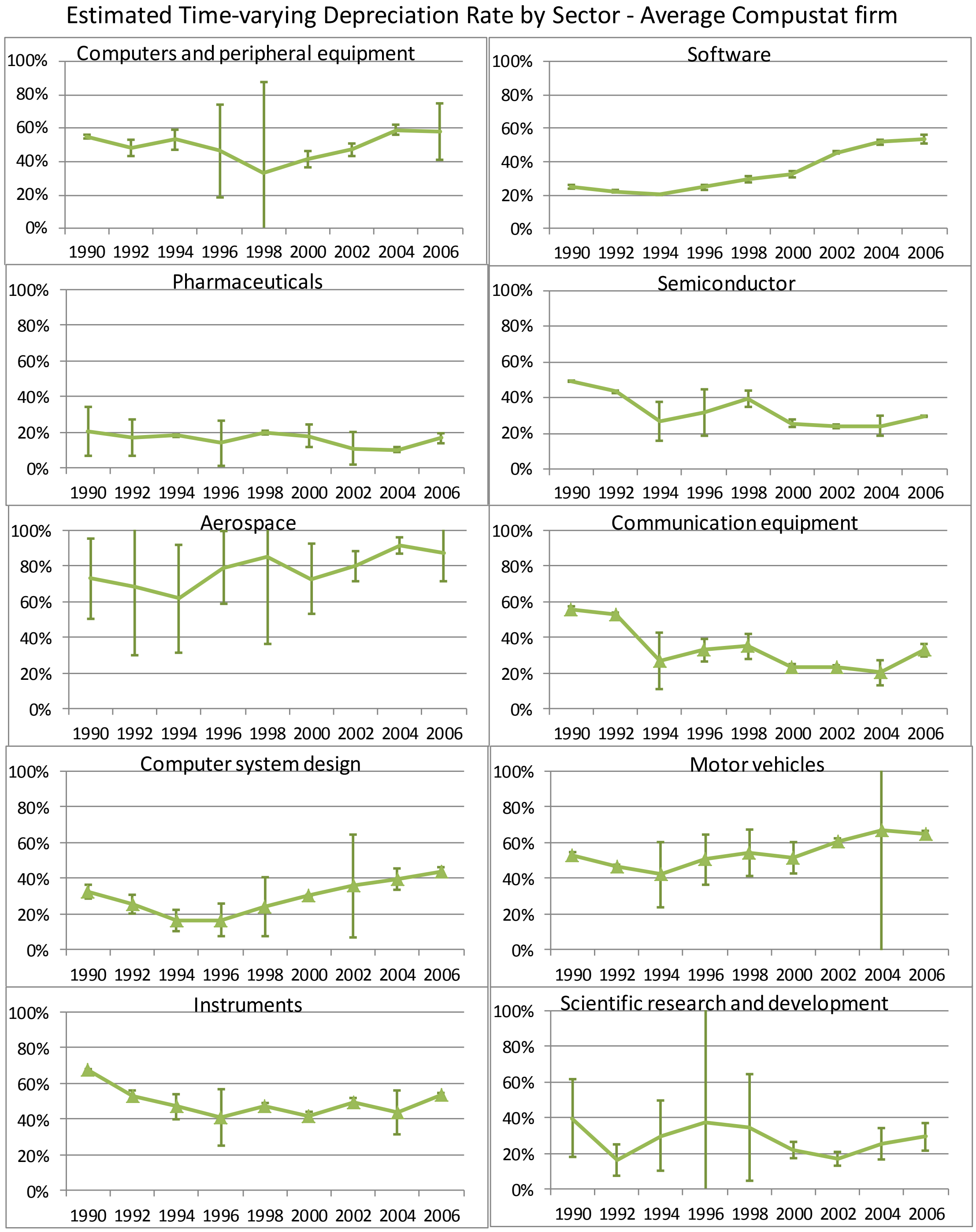


Figure 4: Time-varying R&D Depreciation Rates Based on Compustat data



Appendix A: Nonlinear GMM

The GMM estimatior is well-known. In this appendix we outline its implementation for our model. Let $m(\theta) \equiv z_i\varepsilon_i$ and $s(\theta) \equiv E[m(\theta)m(\theta)']$. The corresponding analog sample moments are:

$$m(\theta) = \frac{1}{n-1}\sum_{i=2}^{n}\begin{pmatrix} \dfrac{(1+G)^i}{I_\Omega}\cdot\theta_0 \exp\left[\dfrac{R_i}{\theta_0(1+G)^i}\right] - \dfrac{q_i(1+g)^d}{(1+r)^{d-1}(r+\delta-g)} \\ R_i\cdot\dfrac{(1+G)^i}{I_\Omega}\cdot\theta_0 \exp\left[\dfrac{R_i}{\theta_0(1+G)^i}\right] - \dfrac{q_i(1+g)^d}{(1+r)^{d-1}(r+\delta-g)} \\ q_i\cdot\dfrac{(1+G)^i}{I_\Omega}\cdot\theta_0 \exp\left[\dfrac{R_i}{\theta_0(1+G)^i}\right] - \dfrac{q_i(1+g)^d}{(1+r)^{d-1}(r+\delta-g)} \end{pmatrix}$$

And their variance estimate $s(\theta) = \dfrac{1}{n-1}\sum_{i=2}^{n} m(\hat{\theta})m(\hat{\theta})'$.

Define $\theta \equiv [\theta_0\ \delta]'$ and

$$M(\theta) \equiv \frac{\partial m(\theta)}{\partial \theta} = \begin{bmatrix} \dfrac{\partial m_1}{\partial \theta_0} & \dfrac{\partial m_1}{\partial \delta} \\ \vdots & \vdots \\ \dfrac{\partial m_5}{\partial \theta_0} & \dfrac{\partial m_5}{\partial \delta} \end{bmatrix}$$

The corresponding sample analog is:

$$M(\theta) \equiv \frac{1}{n-1}\sum_{i=2}^{n}\begin{bmatrix} m_{11}(\hat{\theta}) & m_{12}(\hat{\theta}) \\ \vdots & \vdots \\ m_{51}(\hat{\theta}) & m_{51}(\hat{\theta}) \end{bmatrix}$$

Note that GMM estimators are asymptotic normal:

$$\theta \sim N\left(\theta, (n-1)^{-1}V\right)$$

where $V = \left[M^{'} s^{-1} M \right]^{-1}$.

To derive the optimal solution for θ, we solve the following optimization problem using the usual nonlinear minimum distance methods with the initial weight matrix as an identity matrix:

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \left[m(\theta)^{'} \hat{W}_K m(\theta) \right]$$

We continue the iterative operations until the change of the value of the objective function stabilizes.

Appendix B: Additional tables and figures

Figure B-1

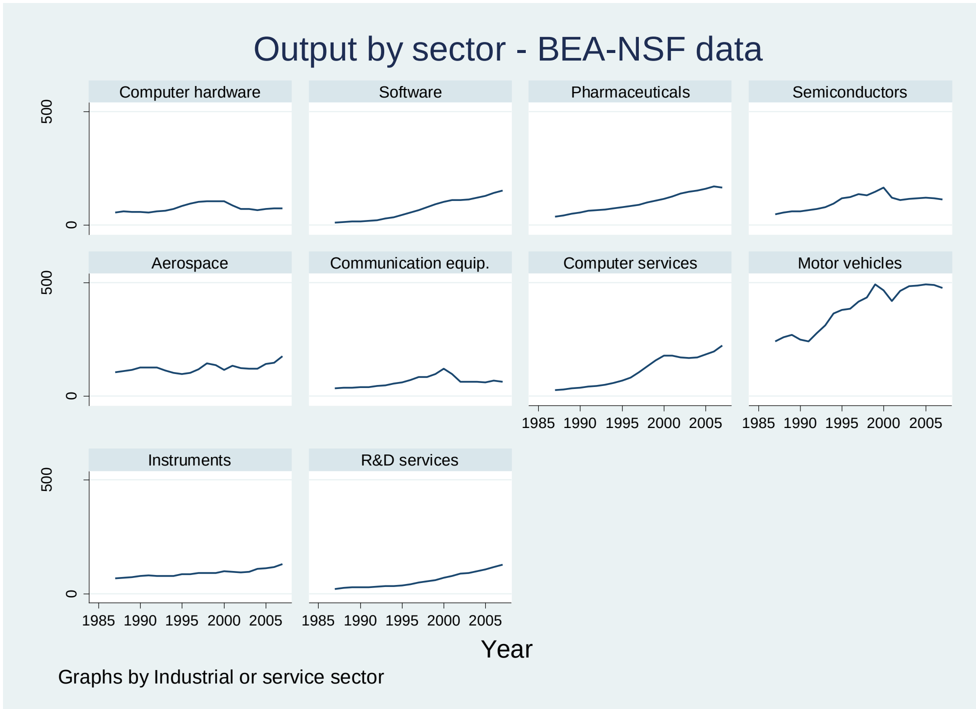


Figure B-2

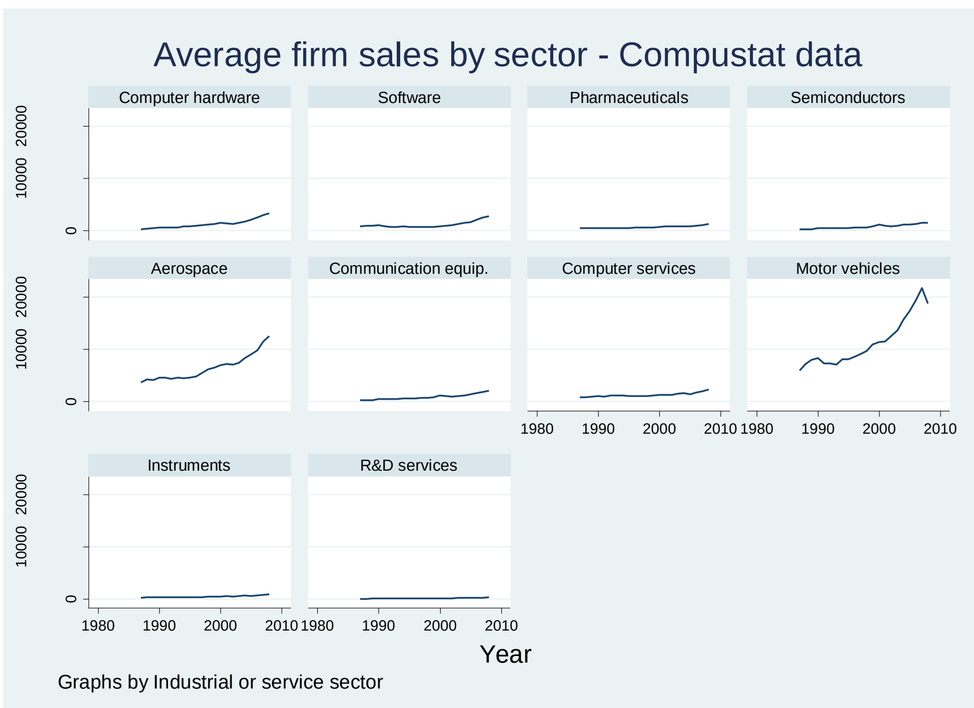


Figure B-3

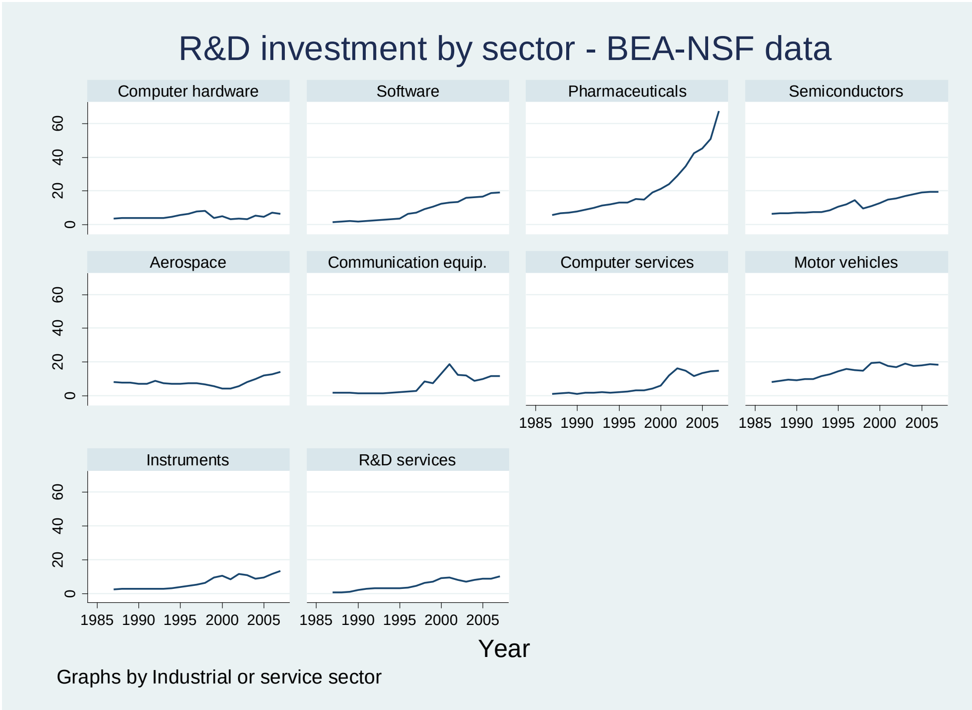


Figure B-4

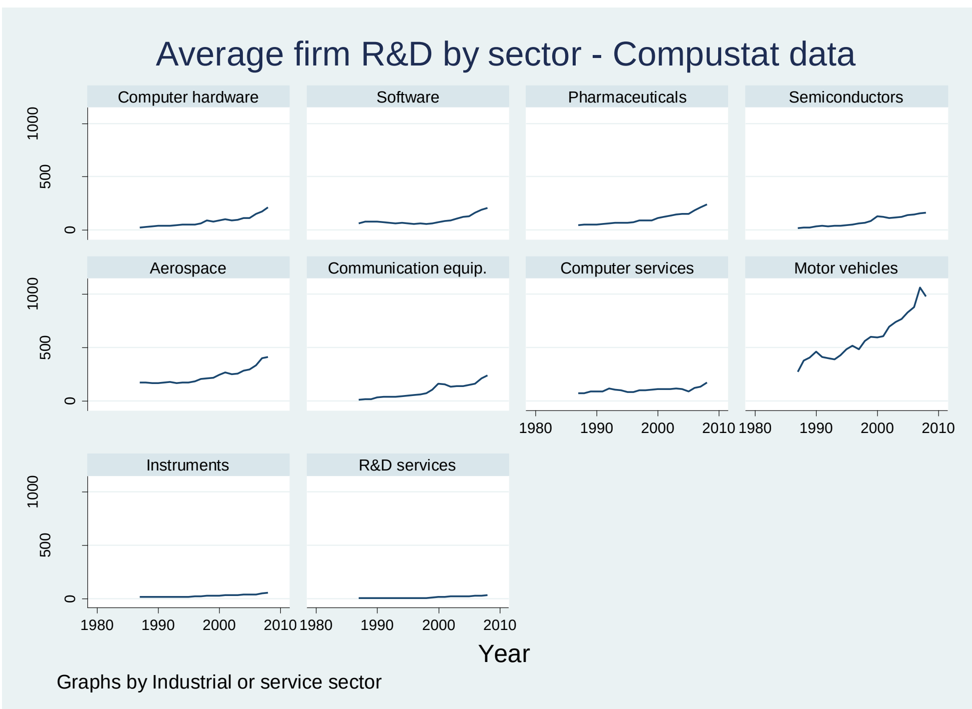
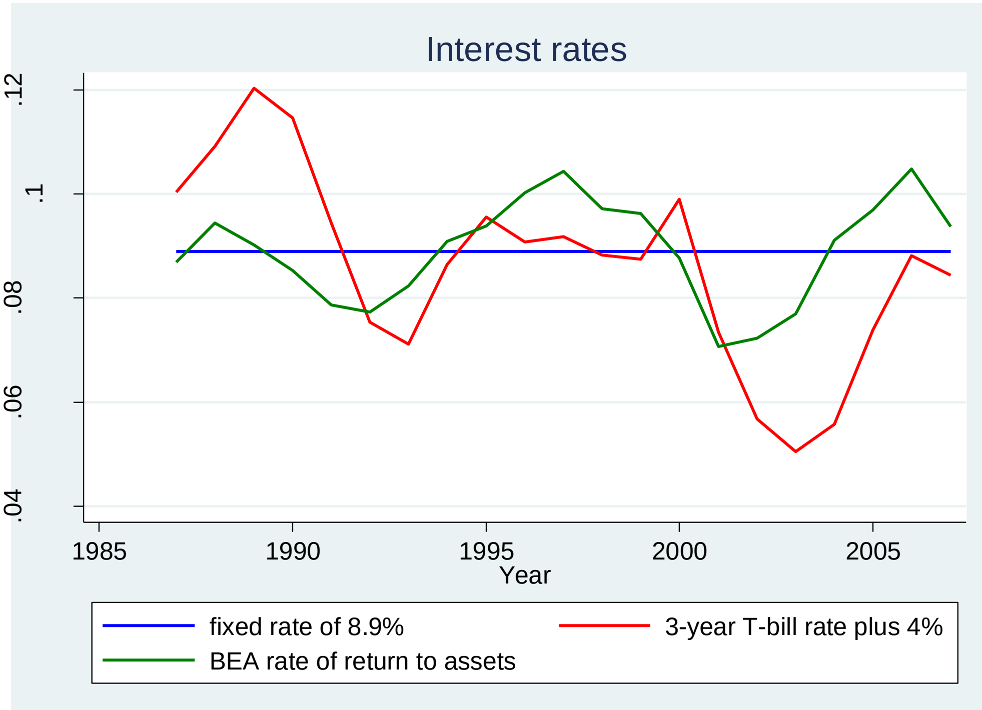


Figure B-5



Interest rates
.04
.06
.08
.1
.12
1985
1990
1995
2000
2005
Year
fixed rate of 8.9%
3-year T-bill rate plus 4%
BEA rate of return to assets